PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 2407–2418

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Analytical and numerical investigation of the characteristics of a soil heat exchanger for ventilation systems

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Received 2 June 2000

Abstract

A mathematical model is developed for calculating the temperature of the soil and air in a soil heat exchanger for ventilation systems. The model is based on the representation of temperature in the form of the Fourier integral. For high-frequency components with characteristic times of the order of 24 h an exact analytical solution is used. Calculation of low-frequency components with characteristic times of the order of a year is based on simulation of a tube by a linear heat source. The degree of decrease in the efficiency of the heat exchanger with decrease in the spacing between its tubes is evaluated. The dependences of the thermal power of the system on the length and diameter of the tubes, depth of their burial and air flow rate are calculated. An analytical expression is obtained for the optimum length of the tube. The evolution of the thermal power of the system during its operation for 10 years only in the winter period is calculated. The results of calculations are compared with experimental data. The procedure developed does not require cumbersome calculations and can be used for working out design recommendations.

1. Introduction

One of the means of reducing expenditures of energy for sustaining the required microclimate in residential buildings, greenhouses, vegetable storehouses and industrial rooms consists in the use of the thermal energy of the soil of the surface layers of the Earth. A soil heat exchanger is a system of tubes buried in soil at a depth of 2–5 m through which outer air is pumped before its entry into the ventilation system. The temperature of the soil at these depths varies little and lies within 2–10 °C. This makes it possible to warm up air in winter and to cool it in summer [1,2]. Experimental soil heat exchanger–storage systems are used in many countries [1–6]. However, despite the practical importance of the problem, the number of works devoted to the development of engineering methods of calculation of the output air temperature and of the temperature field in soil at different time instants is very few. When air passes through tubes, the soil changes its temperature, and this, in turn, influences the air temperature, therefore it is necessary to carry out simultaneous calculation of the soil temperature and of the air temperature in a tube. The majority of the well-known approaches are based on numerical integration of the equation of heat conduction in soil [4,6–10]. To be implemented, they require large expenditures of computational time. Thus, for example, in [4] calculation of one variant of the operation of a system for 3 days took from 20 to 100 min on IBM SP2. These methods are not always suitable for engineering calculations, especially in those cases where the time of operation of a system is of the order of several years, or where many variants are to be considered, and an optimum variant is to be chosen.

In the present work we consider a rapid and relatively simple, and at the same time rather rigorous, method of evaluating the temperature of the air leaving a soil heat exchanger; this method allows one to find the characteristics of the soil heat exchanger with relatively small computational expenditures.

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0017-9310/02/$ - see front matter
PII: S0017-9310(01)00319-2

Nomenclature

c	specific heat of ground
c_a	specific heat of air
d	space between tubes
G	unperturbed temperature of soil
h	ratio of heat exchange coefficient at the soil–atmosphere interface to thermal conductivity of soil
K_0	modified Bessel function of the second kind
l	radius of thermal effect, $l \sim \sqrt{2\chi/\omega}$
L	length of a tube
L_{sat}	length of a tube at which the amplitude of air temperature vibration is decreased e times in comparison with the initial one
$\bar{n}$	unit vector of external normal to the tube surface
Nu	Nusselt number
Pr	Prandtl number
q	ratio of heat flux from tube length unit to heat capacity of soil volume unit
$r = \sqrt{(x-x_0)^2 + (z-z_0)^2}$	distance from tube axis to the point considered
R	tube radius
Re	Reynolds number
S	function defined by Eq. (45)
t	time
T	temperature of soil
T_a	air temperature averaged over tube section
T_w	tube wall temperature averaged over its perimeter
T^*	equivalent temperature of outer air
$\bar{T}$	mean annual temperature of outer air and soil
v	air velocity averaged over tube section
x	Cartesian coordinate oriented horizontally, perpendicularly to the register of tubes
y	Cartesian coordinate oriented horizontally in parallel with the register of tubes in the direction of air flow
z	Cartesian coordinate directed vertically downward

Greek symbols

α	ratio of the coefficient of heat exchange between air and a tube to heat capacity of soil volume unit
β	reciprocal length of temperature relaxation of air in a tube defined by Eq. (7)
χ	thermal diffusivity of soil
δ	tube wall thickness
ρ	soil density
ρ_a	air density
ε	function defined by Eq. (14)
ϕ	phase of oscillations of unperturbed soil temperature
φ	phase shift in air temperature oscillations over length of a tube
γ	increment of decay in air temperature oscillations
η	excess of tube wall temperature over the temperature of nonperturbed ground at the depth of tube burial, $\eta = T_w - G(t, z_0)$
λ_a	thermal conductivity of air
λ_w	thermal conductivity of tube material
ξ	angle between the direction to the point considered and z-axis
ϑ	excess of air temperature over the temperature of nonperturbed ground at the depth of tube burial, $\vartheta = T_a - G(t, z_0)$
ω	circular frequency
Ω	circular frequency
Ψ	phase of temperature oscillations
Δ	Laplace operator
ΔT_1	amplitude of daily fluctuations of input air temperature
ΔT_2	amplitude of seasonal fluctuations of input air temperature
ΔT_3	amplitude of temperature fluctuations defined by Eq. (57)

Subscripts

a	refers to air
g	refers to ground
w	refers to tube wall
ω	denotes Fourier component
0	refers to tube axis
1	denotes daily period
2	denotes annual period

2. Governing equations

Let us introduce a rectangular coordinate system with the origin on the surface of the ground with the z-axis directed into the ground and y-axis parallel to horizontally laid tubes. We assume that the ground is characterized by homogeneous time-independent thermophysical properties. The considered system of equations includes the heat conduction equation

$$\partial T/\partial t = \chi \Delta T \tag{1}$$

with the boundary condition on the ground surface, approximately taking into account radiative and convective mechanisms of heat exchange with the atmosphere and also heat losses for moisture evaporation [11]:

$$\partial T/\partial z|_{z=0} = h(T - T^*(t))|_{z=0}, \tag{2}$$

where h is the ratio of the generalized coefficient of heat exchange to the thermal conductivity of soil, $T^*(t)$ is the equivalent air temperature. On the walls of the tubes the heat flux to the soil is prescribed:

$$-\chi(\vec{\nabla}T \cdot \vec{n})|_{\text{wall}} = q(t, y)(2\pi R)^{-1}, \tag{3}$$

where $\vec{n}$ is the outer normal to the surface of a tube, R is its radius, ρcq is the heat flux from the tube unit length:

$$q(t, y) = \alpha[T_a(t, y) - T_w(t, y)], \tag{4}$$

where $T_a(t, y)$ is the tube section-mean air temperature, $T_w(t, y)$ is the tube wall temperature mean over its perimeter. It follows from Eqs. (3) and (4) that a heat flux from each of the tubes is isotropic (constant over the tube perimeter). This assumption can be violated in the case of shallow burials and when the spacing between the tubes is small. Evaluations from the formulas given in [12] show that with the diameters of tubes of 0.1–0.3 m and burial depths of 2–5 m, which are typical of soil heat exchangers, the nonisotropicity of the heat flux from a solitary tube can be neglected. As regards the account for the finiteness of spacing between tubes, the approximation of the isotropy of the heat flux makes it possible to evaluate only the initial stage of thermal mutual effect of tubes, when the violation of the isotropicity is still small.

The equation for the air temperature can be represented in the form

$$\partial T_a(t, y)/\partial y = -(\beta/\alpha)q(t, y). \tag{5}$$

The coefficients α and β have the following forms:

$$\alpha = \{\rho c/\pi Nu\lambda_a + \delta\rho c/2\pi R\lambda_w\}^{-1}, \tag{6}$$

$$\beta = Nu\chi_a/R^2 v, \tag{7}$$

where v is the rate of pumping, δ is the tube wall thickness, λ is the thermal conductivity (the subscripts a and w denote air and the wall of the tube), Nu is the Nusselt number, which in the case of turbulent flow is approximately equal to [13]

$$Nu = 0.023 Re^{0.8} Pr^{0.4},$$

where Re and Pr are the Reynolds and Prandtl numbers.

We will seek the temperature of the ground in the form of the sum of the unperturbed temperature of the ground G and the perturbation U caused by the effect of the heat exchanger:

$$T = G + U, \tag{8}$$

where G obeys the equation

$$\partial G/\partial t = \chi\partial^2 G/\partial z^2 \tag{9}$$

subject to the boundary condition

$$\partial G/\partial z|_{z=0} = h(G - T^*(t))|_{z=0}. \tag{10}$$

Substituting Eq. (8) into Eqs. (1)–(5) and taking into account Eqs. (9) and (10), we obtain the equation for the perturbed temperature of the ground:

$$\partial U/\partial t = \chi\Delta U \tag{11}$$

with the boundary conditions on the ground surface:

$$\partial U/\partial z|_{z=0} = hU|_{z=0} \tag{12}$$

and on the walls of the tubes:

$$-\chi(\vec{\nabla}U \cdot \vec{n})|_{\text{wall}} = q(t, y)(2\pi R^{-1}) + \varepsilon, \tag{13}$$

where

$$\varepsilon = \chi(\vec{\nabla}G \cdot \vec{n})|_{\text{wall}}, \tag{14}$$

$$q(t, y) = \alpha[\vartheta(t, y) - \eta(t, y)], \tag{15}$$

$\vartheta(t, y)$ and $\eta(t, y)$ are the excesses of the temperatures of air and of the tube wall over the unperturbed soil temperature at the depth of tube burial. The equation for the air temperature is

$$\partial\vartheta(t, y)/\partial y = -(\beta/\alpha)q(t, y). \tag{16}$$

The solution of Eq. (9) subject to boundary condition (10) leads to the following expression for the unperturbed temperature of the ground [14]:

$$G(t, z) = (2\pi)^{-1}\int G_\omega(z)\exp(i\omega t)\,d\omega, \tag{17}$$

where

$$G_\omega(z) = G_\omega(0)\exp\left(-z\sqrt{\omega/2\chi}\right)\exp\left(-iz\sqrt{\omega/2\chi} - i\phi_\omega\right), \tag{18}$$

$G_\omega(0)$ and ϕ_ω are the amplitude and phase of the ground surface temperature fluctuations:

$$G_\omega(0) = T^*_\omega h\left[\left(h + \sqrt{\omega/2\chi}\right)^2 + \omega/2\chi\right]^{-(1/2)}, \tag{19}$$

$$\phi_\omega = \operatorname{arctg}\left[\sqrt{\omega/2\chi}\left(h + \sqrt{\omega/2\chi}\right)^{-1}\right], \tag{20}$$

T^*_ω is the Fourier-component of the temperature $T^*(t)$.

3. Method of calculation

The system of Eqs. (11)–(16) has no analytical solution. A large number of approximate methods of solving the system of Eqs. (11)–(15), used for calculating the nonstationary temperature field in the soil at a given air temperature in the tube, were developed in studying the thermal conditions of oil- and gas-pipes [12,15–20]. For the case of large depths of tube burial, where the effect of the ground–atmosphere interface and nonuniformity of the unperturbed temperature of the ground can be neglected, an exact solution of the system of Eqs. (11), (13)–(16) was obtained for $\varepsilon = 0$ in [21] by the Laplace transformation method. However, its numerical implementation turned out to be rather complex and approximate methods of solving this problem were suggested [22,23].

In the present work, the calculation of the temperature of the air leaving the heat exchanger is carried out with the aid of the Fourier transformation method. Generally speaking, the Fourier transformation method can be applied for calculating the characteristics of a heat exchanger at any times of the beginning of the operation of the system and in the case of an arbitrary dependence of the input air temperature on time. But it is especially convenient for engineering evaluations of the efficiency of the system at the stage of design. The thing is that in the Fourier-spectra of the fluctuations of both the temperature of the unperturbed soil and the temperature of the atmospheric air entering the heat exchanger, there are two intense maxima at the frequencies of the order of reciprocal 24 h and reciprocal year with a trough between them. Therefore, at the stage of design it is often sufficient to represent the temperature of the soil and the temperature of the input air in the form of the sum of two sinusoids with the daily and annual periods and to limit ourselves to the consideration of the steady regime of operation of the heat exchanger. In this case, the temperature of the air leaving the heat exchanger is also the superposition of two harmonics with the amplitudes and phases which are to be determined.

In the case of the arbitrary dependence of the input air temperature on time we will subdivide conventionally the air temperature and the perturbed temperature of the soil into high- and low-frequency components with characteristic timescales of 24 h and min (t, year), where t is the time of operation of the system ($t \gg 24$ h). This division is associated with different means of evaluation of the Fourier-components for different frequencies. For high-frequency Fourier components an exact solution is formed, whereas for the region of low frequencies an approximate expression is found which is based on the model of a linear source [14].

3.1. High-frequency component

The radius of the effect, $l \sim \sqrt{2\chi/\omega}$, of the heat source that sends into the soil a heat flux that depends harmonically on time and has the frequency $\omega = \Omega_1 = 7.3 \times 10^{-5}\ \mathrm{s}^{-1}$ that corresponds to the daily period is approximately 7–15 cm for typical soils with $\chi = (2.5\text{–}10) \times 10^{-7}\ \mathrm{m}^2/\mathrm{s}$ [24]. The indicated value is much lower for both typical depths of burial and lengths of pipes (more than 10 m) and the spacing between them (1–2 m). This makes it possible to ignore the thermal interaction of the pipes and also the influence of the ground–atmosphere interface. Moreover, it follows from Eqs. (18)–(20) that at the depths exceeding 1 m, when $\omega \approx \Omega_1$, it is possible to assume that $G_\omega = 0$. As a result, we arrive at the problem of a single tube in an infinite uniform space. Moreover, we may assume that the air temperature in the tube and, consequently, also the temperature of the soil do not change in the direction of the y-axis at the distances of the order of the influence radius l. Therefore, it is possible to ignore the longitudinal (along the tube) heat flux in the soil in comparison with the transverse one. We also assume that the pumping rate is independent of time. Taking into account the foregoing and going over to a cylindrical coordinate system, we obtain from Eqs. (11), (13)–(15) the following system of equations for the high-frequency components of the soil and air temperatures for each of the tubes:

$$\mathrm{i}\omega U_\omega = \chi \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial U_\omega}{\partial r}, \tag{21}$$

$$-\chi \partial U_\omega/\partial r|_{r=R} = q_\omega(y)(2\pi R)^{-1}, \tag{22}$$

$$q_\omega(y) = \alpha[\vartheta_\omega(y) - \eta_\omega(y)], \tag{23}$$

$$\partial\vartheta_\omega(y)/\partial y = -(\beta/\alpha) q_\omega(y). \tag{24}$$

The solution of Eq. (21) with account for condition (22) for the Fourier-component of the soil temperature at the distance r from the tube axis has the form

$$U_\omega(r,y) = -q_\omega(y)(2\pi\chi)^{-1} K_0\left(\sqrt{\mathrm{i}}D\right)\left[\left(\sqrt{\mathrm{i}}P\right)\cdot K_0'\left(\sqrt{\mathrm{i}}P\right)\right]^{-1}, \tag{25}$$

where K_0 is the modified Bessel function of the second kind, R is the tube radius

$$D = r \cdot \sqrt{\omega/\chi}, \quad P = R \cdot \sqrt{\omega/\chi}. \tag{26}$$

Substituting $U_\omega(R, y) = \eta_\omega(y)$ into Eq. (23), we find the coupling between the heat flux and the air temperature:

$$q_\omega(y) = \alpha(1 + S_\omega)^{-1}\vartheta_\omega(y), \tag{27}$$

where

$$S_\omega = -\alpha(2\pi\chi)^{-1} K_0\left(\sqrt{\mathrm{i}}P\right)\left[\left(\sqrt{\mathrm{i}}P\right)\cdot K_0'\left(\sqrt{\mathrm{i}}P\right)\right]^{-1}. \tag{28}$$

As a result, we arrive at the following equation for the Fourier-component of the temperature of the air that in the tube covered the path of length y:

$$\partial\vartheta_\omega(y)/\partial y = -\beta(1 + S_\omega)^{-1}\vartheta_\omega(y). \tag{29}$$

Its solution has the form

$$\vartheta_\omega(y) = \vartheta_\omega(0)\exp[-(\gamma_\omega + \mathrm{i}\varphi_\omega)y], \tag{30}$$

where

$$\gamma_\omega = Re[\beta(1+S_\omega)^{-1}], \quad \varphi_\omega = -\mathrm{Im}[\beta(1+S_\omega)^{-1}], \tag{31}$$

The temperature itself is defined by the expression

$$\vartheta(t,y) = (2\pi)^{-1}\int \vartheta_\omega(y)\exp(\mathrm{i}\omega t)\,\mathrm{d}\omega. \tag{32}$$

In particular, if air of temperature

$$\vartheta(t,0) = \Delta T \cdot \cos\Omega t \tag{33}$$

is supplied to the tube inlet beginning from the zero time instant, then after the time of the order Ω^{-1} steady operational conditions are developed, and the temperature of the air that in the tube covered the path of length y is defined by the expression

$$\vartheta(t,y) = \Delta T\exp(-\gamma_\Omega y)\cos(\Omega t + \varphi_\Omega y). \tag{34}$$

Similarly, an expression for the temperature of the soil can be obtained with the aid of Eqs. (25), (27) and (30).

The aforegoing solution makes it possible to evaluate the accuracy of the approximation of a volumetric source which is often used for calculating the temperature of the soil. Its essential idea is that the region occupied by the tube is replaced by the soil in which there are volumetric heat sources with the power equal to that emitted by the tube. The wall temperature is identified with the temperature of the soil at the boundary of the region indicated. The situations were considered when heat release was concentrated entirely on the straight line coinciding with the tube axis (linear source [14]) and on the cylindrical surface coinciding with the tube surface (cylindrical surface source [14]). Analytical solutions for these cases differ from the solution given for the tube only by the form of the expressions for S_ω (28). For the model of a linear source

$$S_\omega = (\alpha/2\pi\chi)K_0\left(\sqrt{\mathrm{i}}P\right). \tag{35}$$

This expression can be obtained similarly to Eq. (28) if in Eq. (25) we assume that $R \to 0$ and take into account that [25]

$$zK_0'(z) \to -1 \quad \text{when } z \to 0. \tag{36}$$

For the model of a cylindrical source, which is the superposition of linear sources on a cylindrical surface of radius R, we have

$$S_\omega = (\alpha/2\pi\chi)K_0\left(\sqrt{\mathrm{i}}P\right)I_0\left(\sqrt{\mathrm{i}}P\right), \tag{37}$$

where I_0 is the modified Bessel function.

As seen from Fig. 1, in the case of harmonic fluctuations of the input temperature with the daily period the wall temperature amplitudes calculated by the model of the linear and cylindrical source, differ from the exact one by 30% for a tube of diameter 23 cm, however, the

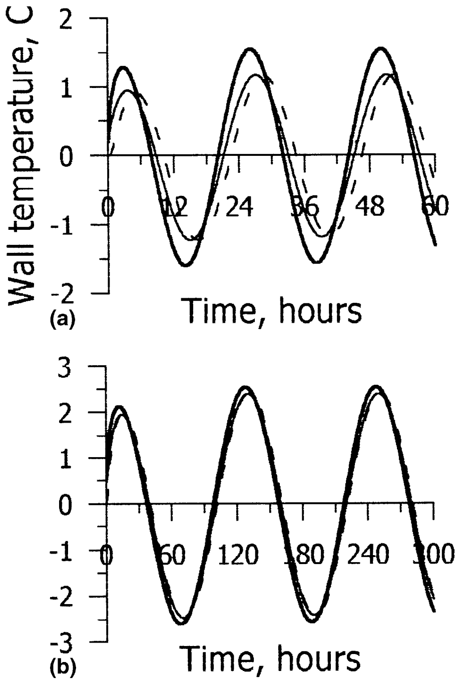

Fig. 1. Excess of the tube wall temperature over the background temperature of soil at a distance of 23 m from the entrance. The tube diameter is 23 cm, air flow rate 407 m^3/h. The temperature of the entering air was prescribed by formula (33) with $\Delta T = 10$ °C and the frequency Ω corresponding to the period equal to (a) a day, (b) 5 days. Solid line, exact calculation by (28), (30)–(32); solid thin line, the model of the cylindrical source; dashed line, of linear one.

model of the linear source leads to a larger shift of the phase relative to the exact solution. We note that the model used in numerical calculations of daily variations of temperature and which represents a source uniformly distributed over the tube section [4] must give an error that lies between the above considered extreme cases of the models of the linear and cylindrical source. The higher the accuracy of the model approximations is the more slowly the input temperature changes and the smaller is the diameter of the tube. As seen, in particular, from Fig. 1(b), the accuracy is equal to several percent for characteristic times of the change in the input temperature of air of the order of several days for the diameters of tubes of the order of 0.1–0.3 m. Therefore, the low-frequency components of the temperature with characteristic times much longer than 24 h can be considered in the approximation of the volumetric, in particular, linear or cylindrical source.

3.2. Low-frequency component

The radius of the influence $l \sim \sqrt{2\chi/\omega}$ at the frequency $\omega = \Omega_2 = 2\times10^{-7}$ s^{-1} corresponding to the yearly period is equal approximately to 1.5–3 m. This distance is commensurable with the depths of burial of tubes and may considerably exceed the distance between them. Therefore, in the case of low frequencies the thermal interaction of tubes may turn to be substantial. Moreover, in calculation of the low-frequency components of the temperature of the soil, especially of its surface layers, it is generally necessary to take into account boundary condition (12) on the ground surface. However, it is shown below that the influence of the ground–atmosphere on the thermal power of the system for the conditions of interest for us is usually small.

One other factor, a priori insignificant in the region of high frequencies but requiring evaluation in the case of low frequencies is the nonuniformity of the unperturbed temperature of the ground that leads to the appearance of the additional term (14) in boundary condition (13). We will consider for simplicity a heat exchanger consisting of one tube. From expression (13) it is seen that the influence of the unperturbed soil temperature nonuniformity is equivalent to a certain additional heat flux from the tube. Evaluating the integral from the heat flux through the surface of unit length of the tube with the aid of the Ostrogradskii–Gauss theorem and thereafter using the heat conduction equation, we find

$$\delta q_\omega \sim \int \chi[\vec{\nabla} G \cdot \vec{n})|_{\text{wall}}\,\mathrm{d}S = \chi \int \Delta G_\omega\,\mathrm{d}V \approx \mathrm{i}\omega G_\omega \pi R^2. \tag{38}$$

Substituting expression (18) into (38), we obtain

$$\delta q_\omega \sim \mathrm{i}\omega\pi R^2 G_\omega(0) \exp\left\{-(1+\mathrm{i})z_0\sqrt{\omega/2\chi} - \mathrm{i}(\phi_\omega - \pi/4)\right\}. \tag{39}$$

To evaluate the heat flux, we assume that the tube is long enough, so that it is possible to neglect the longitudinal component of flow at a rather large depth z_0, so that conditions on the surface of the soil do not influence heat exchange between the air and the soil. In this case, the results of the previous section are valid and, as seen from (27) and (35), the flux can be presented in the form

$$q_\omega = \alpha\vartheta_\omega / \left[1 + (\alpha/2\pi\chi)K_0\left(R\sqrt{\mathrm{i}\omega/\chi}\right)\right]. \tag{40}$$

Taking into account that [25]

$$K_0(z) \to -\ln(z/2) \quad \text{for } z \to 0, \tag{41}$$

we obtain

$$|\delta q_\omega / q_\omega| \sim |G_\omega(0)/\vartheta_\omega| \left[(2\pi\chi/\alpha) - \ln\left(R\sqrt{\omega/4\chi}\right)\right] \times R^2(\omega/2\chi)\exp\left\{-z_0\sqrt{\omega/2\chi}\right\}. \tag{42}$$

This quantity may serve as a measure of the error in calculation of the thermal power of the system introduced by the ignorance of the nonuniformity of the unperturbed temperature of the soil. Calculations show that the indicated error increases with increase in the tube diameter and decrease in the rate of pumping. In one of the most unfavorable cases with $v = 0.5$ m/s and $2R = 0.5$ m for $\omega = \Omega_2$ this error at $|G_\omega(0)/\vartheta_\omega| = 1$ is approximately equal to 3% at a depth of 1.5 m and decreases exponentially with increase in the depth of burial. Thus, calculation of the thermal power of the soil heat exchanger can be performed with sufficient accuracy without account for the nonuniformity of the unperturbed temperature of the soil by discarding the second term on the right-hand side of (13).

Now, let us pass to the evaluation of the ground–atmosphere interface. As shown above, the low-frequency components of the temperature with characteristic times much greater than 24 h can be calculated in the approximation of a volumetric surface. The use of the indicated approximation makes it possible to reduce the problem (11)–(13) with a boundary condition on the tube surface to Eq. (11) modified by addition of a volumetric source, and to the boundary-value condition (12). This, in turn allows one to obtain an expression for a low-frequency component of the temperature of the soil with the aid of the Green's function in quadratures and to avoid numerical integration of the heat conduction equation. In this case boundary condition (12) can be taken into account by the method of fictitious sources [26]. The model of a linear source for the case of a single tube leads to the following expression for the soil temperature exceeding its background value:

$$\begin{aligned} U(x,y,z,t) &= \int_{-\infty}^{\mathrm{t}} \mathrm{d}\tau[4\pi\chi\cdot(t-\tau)]^{-3/2}\int_0^{\mathrm{L}} \mathrm{d}y' q(\tau, y') \\ &\cdot \exp\left[-(y-y')^2[4\chi\cdot(t-\tau)]^{-1}\right] \\ &\times \exp\left[-r^2[4\chi\cdot(t-\tau)]^{-1}\right] \\ &\times \left\{1 + \exp\left[-(rz + 4zz_0)[4\chi\cdot(t-\tau)]^{-1}\right](1 - 2\cdot S)\right\}, \end{aligned} \tag{43}$$

where

$$r^2 = (x - x_0) + (z - z_0), \tag{44}$$

$$S = \int_0^\infty \mathrm{d}p \cdot \exp\{-p - [2(z+z_0)ph + p^2][4\chi h^2\cdot(t-\tau)]^{-1}\}. \tag{45}$$

Expression (43) takes into account both the transverse and longitudinal heat fluxes. It is valid at arbitrary lengths of tubes and takes into account the boundary condition (12) on the ground surface.

Expression (43) at $r = R$ for the tube wall temperature together with equality (15) and the equation for the air temperature in a tube (16) forms the initial system of equations for a low-frequency component of the temperature of air and soil. Its numerical solution can be performed with relatively large time-steps, and this allows one to calculate the evolution of the system over a large interval, up to several years.

The influence of boundary conditions on the thermal power of the soil heat exchanger will be evaluated for two extreme values of the coefficient of heat exchange on the interface between the ground and the atmosphere: $h = 10^{-5}$ m^{-1} and $h = 10^5$ m^{-1}. Fig. 2 presents the dependences of the peak values of thermal power on the burial depth of tubes at the indicated values of the constant h of the interaction of the ground surface with the surrounding medium. The temperature of the air at the inlet changed harmonically with yearly period. It is seen that beginning from the depths of the order of 2 m, the difference between the thermal powers at the extreme values of the constant h becomes negligibly small which means the absence of the dependence of thermal power on the boundary conditions on the ground surface. This means that the influence of fictitious sources on the temperature of the soil in the region of the tube can be neglected assuming the expression in braces in expression (43) equal to unity at the values of τ making the main contribution to the integral. Thus, if the depths of burial of the tubes exceed 2 m, from the viewpoint of calculation of the thermal power of a heat exchanger the soil can be considered as a homogeneous infinite medium with time-dependent unperturbed temperature at the depth of the burial of the tubes. This value can be taken from the data of field measurements. Taking into account boundary condition (12) may turn out to be important for estimating the temperature of the surface layers of the ground. It is also seen from this figure that the maximum heat powers are attained at depths of the order of 5 m.

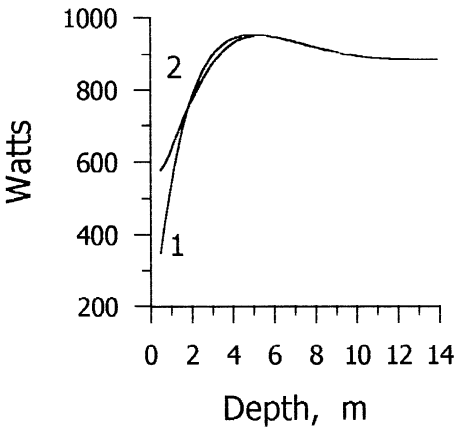

Fig. 2. Amplitude of seasonal average daily fluctuations of the heat power of a heat exchanger depending on the depth of burial of a tube at different values of the heat exchange coefficient on the soil-atmosphere interface: (1) $h = 10^{-5}$; (2) $h = 10^5$ m^{-1}. Tube length 23 m, diameter 0.23 m, air flow rate 1500 m^3/h.

The approach based on the use of expression (43) at rather large burial depths z_0 of the tube (virtually larger than 2 m) naturally agrees with the above-considered case of long tubes $L \gg \sqrt{2\chi/\omega}$. If at the length of approximately $\sqrt{2\chi/\Omega_2}$ the heat flux $q(\tau, y')$ changes little, it can be taken equal to $q(\tau, y)$, i.e., to the value in the maximum over the y' integrand function and for $L \gg \sqrt{2\chi/\omega}$ it can be integrated in (43) over y' in infinite limits. Further in expression (43) we neglect the contribution of fictitious sources and obtain

$$U(x,y,z,t) = \int_{-\infty}^{t} \mathrm{d}\tau [4\pi\chi(t-\tau)]^{-1} q(\tau,y) \exp[-r^2[4\chi(t-\tau)]^{-1}]. \tag{46}$$

Passing in (46) to the integration variable $\xi = t - \tau$ and representing $q(t-\tau, y)$ in the form of the Fourier integral, we find that the determined Fourier component of (46) is equal to

$$U_\omega(r,y) = q_\omega(y)(2\pi\chi)^{-1} K_0(\sqrt{\mathrm{i}}D), \tag{47}$$

With (36) taken into account, this coincides with the limit of low frequencies of expression (25).

Table 1 lists peak values of the low-frequency component of the power as functions of the tube length. The values are calculated for the minimum frequency $\omega = \Omega_2$ in two ways: from (43), (15) and (16) and from (34), (30)–(32), and (35). From the data of the table it follows that calculation of heat exchangers whose tubes are longer than 10 m, i.e., actually with all of the tubes of practical interest can be carried out by formula (34) with an accuracy not worse than 5%. The error decreases with increase in frequency. Thus, when the depths of burials z_0 are larger than 2 m and the tubes are longer than 10

Table 1
Amplitudes of seasonal average daily fluctuations of the heat power of a heat exchanger (W) calculated by different methods as functions of tube length. Tube diameter 0.23 m, air flow rate 407 m^3/h, burial depth 4 m

Tube length, m	2.5	5.0	10.0	15.0	20.0
Calculation by Eq. (43), (15) and (16)	98.1	175.2	319.7	454.4	578.3
Calculation by Eq. (34), (30)–(32) and (35)	80.3	157.7	304.3	440.5	567.2

m, the results obtained for high frequencies in the case of one tube are also valid for low frequencies.

Let us consider now thermal interaction of a system of many tubes using as a basis the model of a linear source and the results obtained for high frequencies. For simplicity we restrict ourselves to the case of an infinitely large set of identical horizontal, parallel d-spaced tubes. Here, the distributions of the heat flux, temperatures of air and walls along the tube length are identical for all the tubes. The heat flux $q/2\pi R$ defined by expression (13) for $\varepsilon = 0$ is assigned on the side surface of each tube. In passing to the model of a linear source we assume that on the axis of the volume of the soil which replaced the tube the heat of power q is evolved, so that in the quasi-stationary regime (for $R\sqrt{2\omega/\chi} \ll 1$) the same heat flux q, as in the case of a tube, emerges through the surface of radius R.

The Fourier-component of the temperature field created by one linear source has the form of (47). According to the superposition principle, the temperature created by many sources is equal to

$$U_\omega(r,y) = q_\omega(y)(2\pi\chi)^{-1} \times \sum_{n=-\infty}^{n=\infty} K_0\left\{\sqrt{\mathrm{i}\omega/\chi}\sqrt{r^2 + (nd)^2 - 2ndr\sin\xi}\right\}, \tag{48}$$

where r is the distance from the axis of the tube with the number $n = 0$ to the considered point in the plane (x,z), ξ is the angle between the direction to the considered point and z-axis. Taking into account the fact that the spacing between the tubes greatly exceeds their radius

$$R \ll d, \tag{49}$$

we substitute the expression for $U_\omega(R,y)$ into (15) and obtain Eq. (29) but with the expression for S_ω differing from that in Eq. (30):

$$S_\omega = (\alpha/2\pi\chi)\left\{K_0\left(R\sqrt{\mathrm{i}\omega/\chi}\right) + 2\sum_{n=1}^{\infty} K_0\left(nd\sqrt{\mathrm{i}\omega/\chi}\right)\right\}. \tag{50}$$

A further derivation of the expression for the temperatures of air and soil is similar to the expressions in Section 3.1. As a result, we obtain that the temperature of the air that passed through the tube the path of length y is determined by expression (30) with the redefined expression for S_ω (50).

3.3. Joining of solutions

Thus, we have expressions (30)–(32) to evaluate the exit temperature of air at all the values of frequencies. They are based on explicit account for the fact that all the tubes are long ($L \gg \sqrt{2\chi/\omega}$) and that from the point of view of calculation of the heat power of the heat exchanger the nonperturbed ground can be considered as a homogeneous infinite medium with the temperature equal to the temperature at the depth of the burial of tubes. The speed of air pumping is independent of time. The quantity S_ω entering into expressions (30)–(32) is defined by expression (28) for high frequencies and by expression (50) for low ones. When natural condition (49) is satisfied, there is an intermediate region of frequencies where both limits are valid, and this ensures smooth joining of solutions. In the intermediate region indicated expressions (28) and (50) virtually coincide; they describe a single tube in the linear source approximation.

In fact, expression (28) is transformed into expression (35), which corresponds to the linear source approximation when

$$\omega < 2\chi/R^2. \tag{51}$$

It is at these frequencies that expression (50) is valid. It is possible to neglect the thermal interaction of tubes and treat them as single ones when

$$\omega > 2\chi/d^2. \tag{52}$$

Consequently, in the range of the frequencies that satisfy the inequality

$$2\chi/R^2 > \omega > 2\chi/d^2, \tag{53}$$

expressions (28) and (50) are joined. Thus, expressions (30)–(32) together with (28) and (50) give the full Fourier-spectrum of the temperature at all the values of frequencies and can be used to calculate the exit temperature and heat power of the heat exchange with tubes at burials deeper than 2 m.

4. Method of rapid evaluation of the air temperature

As a rule, for engineering evaluations at the stage of design, it is sufficient to know only the parameters of the stationary regime for the simplest dependence of the inlet air temperature and nonperturbed temperature of the ground at the depth of the burial of tubes on time:

$$T_\mathrm{a}(t,0) = \bar{T} + \Delta T_1\cos(\Omega_1 t + \Psi_1) + \Delta T_2\cos(\Omega_2 t + \Psi_2), \tag{54}$$

$$G(t,Z_2) = \bar{T} + \Delta T_\mathrm{g}(Z_0)\cos(\Omega_2 t + \Psi_\mathrm{g}(z_0)), \tag{55}$$

where $\bar{T}$ is the mean yearly temperature of the air and ground; $\Delta T_1, \Psi_1$ and $\Delta T_2, \Psi_2$ are the amplitudes and phases of daily and yearly oscillations of the inlet air temperature; ΔT_g and Ψ_g are the amplitude and phase of yearly oscillations of the temperature of the ground at the depth of the burial of tubes. The mean yearly temperatures of air and ground may differ by 1–3 °C [27]. They are taken equal in model (54) and (55). In the stationary regime of the temperature of the air that passed through the tube the path of length y is

$$T_a(t,0) = \bar{T} + \Delta T_g(z_0)\cos(\Omega_2 t + \Psi_g(z_0)) + \Delta T_1 \exp(-\gamma_{\Omega_1} y)\cos(\Omega_1 t + \Psi_1 - \varphi_{\Omega_1} y) + \Delta T_3 \exp(-\gamma_{\Omega_2} y)\cos(\Omega_2 t + \Psi_3 - \varphi_{\Omega_2} y), \quad (56)$$

where

$$\Delta T_3 = \sqrt{\Delta T_2^2 + \Delta T_g^2 - 2\Delta T_g \Delta T_2 \cos(\Psi_2 - \Psi_g)}, \quad (57)$$

$$\Psi_3 = \mathrm{arctg}[(\Delta T_2 \sin\Psi_2 - \Delta T_g \sin\Psi_g)/(\Delta T_2 \cos\Psi_2 - \Delta T_g \cos\Psi_g)]. \quad (58)$$

The quantities γ and φ are determined from formula (31). The quantities S_{Ω_1} and S_{Ω_2}, contained in γ and φ, are calculated with the aid of Eqs. (28) and (50), respectively. The use of formula (56) allows one to evaluate the temperature of the air that leaves the heat exchange at minimum machine time expenditures.

5. Results of calculations

The calculations were performed with the use of Eq. (56) and also on the basis of the system of Eqs. (43), (15), and (16) that allows one to find the mean daily temperature of the escaping air. The temperature of the air entering into the system was simulated by a sum of the mean temperature and two cosinusoids with the daily and yearly periods (54). The temperature of ground calculated from Eqs. (17) and (18) with account for the mean yearly temperature and one harmonic with a yearly period had the form of Eq. (55). The average annual temperatures of air and ground were taken equal to 10 °C, the amplitude of annual variations of the air temperature was assumed equal to 15 °C and of the ground surface to 12 °C. The amplitude of the daily variations of the air temperature was prescribed equal to 7 °C.

In the calculations given below we mainly used the values of the parameters of one of the actual heat exchangers described in the literature [1]: tube length 23 m, diameter 0.23 m, number of tubes 43, the burial depth 4 m, flow rate of air per one tube 407 m^3/h.

Fig. 3 presents the results of calculations from the system of Eqs. (43), (15), and (16) of the average daily exit temperature of continuous heat exchangers consisting of one and of an infinite number of tubes located at a distance of 1 m. It is evident that the influence of the neighboring tubes decreases the difference between the inlet and exit temperatures, i.e., leads to the decrease of the thermal power of the system per tube.

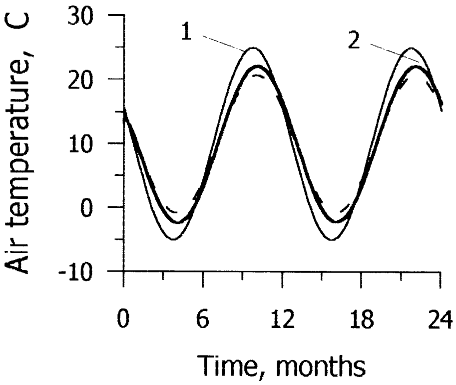

Fig. 3. Average daily temperature of air depending on time: (1) temperature of the air entering the heat exchanger; (2) temperature of the air leaving the heat exchanger that has an infinite number of 1 m spaced tubes. Dashed curve, temperature of the escaping air calculated without account for neighboring tubes. Tube length 23 m, diameter 0.23 m, air flow rate 407 m^3/h, depth of burial 4 m.

The results of the calculations with the use of Eq. (56) that illustrate the degree of the fall of the peak values of the average daily heat power in the stationary regime per tube with a decreasing spacing between them is shown in Fig. 4. These results practically do not depend on the tube diameter. The reduction of the spacing between the tubes to 1.5 m leads, depending on the air flow rate, to a 5–15% loss of the power by the system, while the reduction of the spacing to 1 m leads to a 10–25% loss of the power by the system in comparison with the case of an infinite spacing between the tubes. Since the case of an infinite number of tubes gives the lower boundary of heat power, the power per tube of the system consisting from the finite number of tubes, lies between the cases of one tube and an infinite number of tubes.

Fig. 5 presents the amplitudes of the early and daily fluctuations of the heat power depending on the tube length. For the tube of length 23 m the yearly amplitude of the heat power attains 0.68 kW. The amplitude of daily fluctuations of the power is equal approximately to 0.55 kW, which results in the value of the peak power of the order of 1.2 kW nearly approximating the experimentally obtained [1] maximum heat power of 1.3 kW per tube. The data presented in Fig. 5 show that the thermal efficiency of the ground heat exchanger increases with the tube length attaining saturation at a certain saturation length $L_{sat} = \gamma_\omega^{-1}$ depending on the frequency of oscillations of the inlet temperature. The saturation length characterizes the optimal (from the point of view of heat exchange) length of the tube. Out of the tubes that are much shorter than L_{sat}, air will escape virtually with the same temperature at which it enters. At the lengths of the tubes larger than L_{sat}, the amplitude of the fluctuations of the difference of the

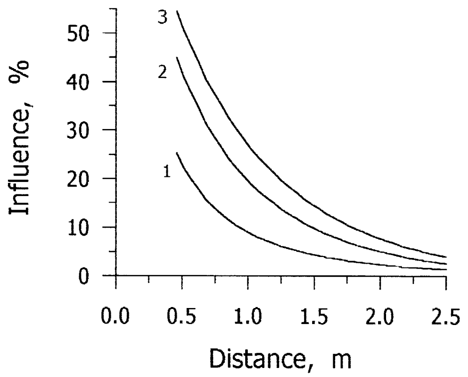

Fig. 4. Percentage of decrease in the heat power of a heat exchanger depending on the space between tubes at different air flow rates: (1) 100 m^3/h; (2) 407 m^3/h; (3) 1500 m^3/h. Tube length is 23 m, diameter 0.23 m; depth of burial 4 m.

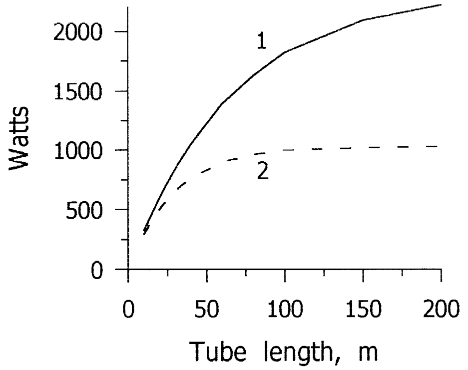

Fig. 5. Amplitude of fluctuations of the heat power of a heat exchanger depending on the length of a tube: (1) seasonal average daily; (2) during 24 h. Tube diameter 0.23 m; air flow rate 407 m^3/h, depth of burial 4 m.

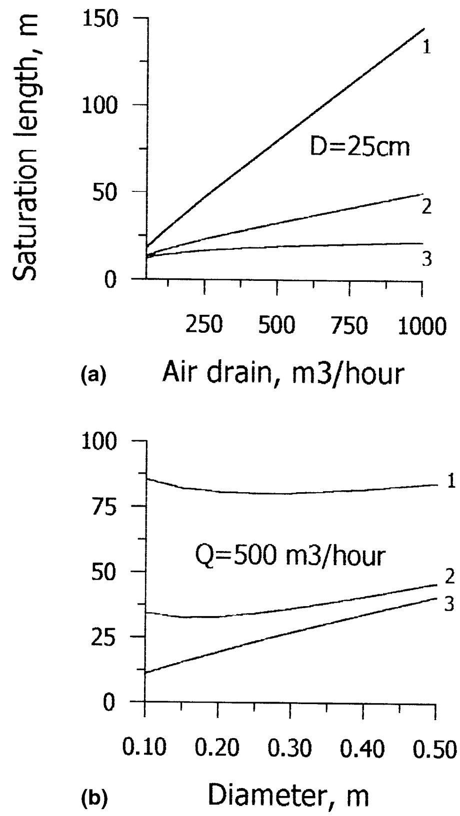

Fig. 6. Dependence of the saturation length (curves 1, 2) and of the characteristic length of thermal interaction (curve 3) on the air flow rate for a tube of diameter 0.25 m (a) and on the tube diameter at the air flow rate equal to 500 m^3/h (b) for the frequency of temperature fluctuations equal to (1) reciprocal year, (2) reciprocal day.

temperatures of the outcoming air and ground close to zero. The temperature of the air leaving the tubes with the length L_{sat} corresponding to the frequency equal to the reciprocal year is nearly the same as the temperature of the nonperturbed ground. The temperature of the air leaving from the tubes of the length L_{sat} that corresponds to the frequency equal to the reciprocal days, will be determined by the average daily temperature of the ground. This temperature depends on the annual component of the outer air temperature and virtually will not experience fluctuations with a daily period. The expression for L_{sat} obtained with (41) taken into account for low frequencies has the form

$$L_{sat} = \beta^{-1} - \pi R^2 v \rho_a c_a (2\pi\rho c\chi)^{-1} \ln[R\sqrt{\omega/4\chi}]. \tag{59}$$

The graphs of the dependence of L_{sat} on the air flow rate and diameter are presented in Fig. 6. From them it follows that L_{sat} increases with increase in the flow rate and at a constant air flow rate does not almost depend on the tube diameter.

From Fig. 6 and Eq. (59) it is seen that the saturation length L_{sat} exceeds the relaxation length β^{-1} over which the air temperature becomes nearly equal to the temperature of the tube wall on condition that the temperature of the wall is fixed. The thing is that with the beginning of the operation of the system the tube wall temperature ceases to be equal to the nonperturbed termperature of the ground and begin to partially follow the air flow temperature. As a result the intensity of heat exchange between the air in the tube and the ground is decreased, and this leads to the increase in the distance at which the amplitude of the air temperature fluctuations may decrease markedly. We note that for a system of many tubes the length of saturation is larger than for a single tube.

Fig. 7 presents the dependence of the amplitude of fluctuations of the average daily heat energy on the air

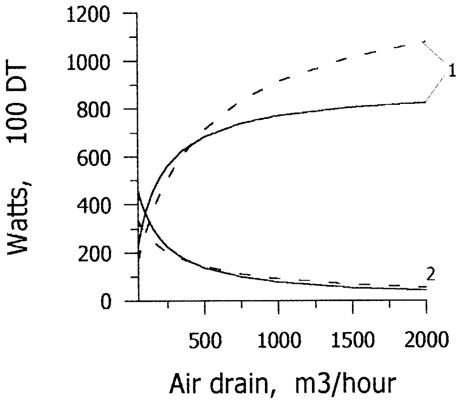

Fig. 7. Dependence of the maximum average daily power (curves 1) and of maximum difference of temperatures (DT) at the inlet and exit of a heat exchanger (curves 2) on the air flow rate. Solid curves, the diameter of the tube is equal to 0.1 m, dashed curve, 0.4 m. The values of the temperature difference were increased 100-fold. The length of the tube is 23 m, the burial depth is 4 m.

flow rate. It is seen that the indicated value increases with the air flow rate and the difference between the inlet and outlet temperature decreases in this case. The variation of the diameter of the tubes from 0.1 to 0.4 m changes the results a little.

Fig. 8 presents the time dependence of the average daily power, calculated from the system of Eqs. (43), (15), (16) per tube for the heat exchanger operating in the warming period from October to March for 10 years. In contrast to the case of continuously operating system, when the loss of heat in winter by the ground is compensated by the pumping in summer, the energy is evacuated in this case only. The amount of the energy pumped out for the season is somewhat decreased with

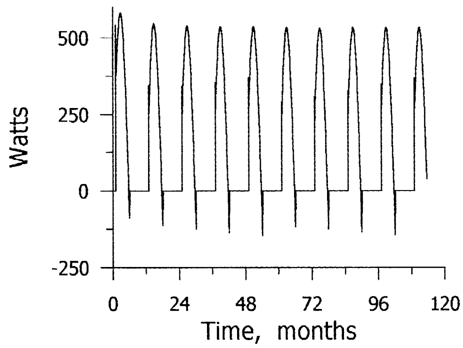

Fig. 8. The time dependence of the average daily power pumped from the soil per one tube. The tubes are spaced 1.5 m apart. The heat exchanger is in operation from October to March. The remaining conditions of calculation are the same as for Fig. 3.

time, but after a period of time of the order of 3–4 years a quasi-stationary regime sets in. It is seen that the heat exchanger with the parameter used here [1] can operate infinitely long without substantial deterioration of its properties.

6. Conclusions

An effective mathematical model has been developed for calculating the temperature of the ground and air in a ground heat exchanger for ventilation systems. The model is based on the representation of temperature in the form of the Fourier integral. For high-frequency components with characteristic times of the order of 24 h an analytical solution has been used. The calculation of the low-frequency components with the characteristic times of the order of a year is based on modelling a tube with a linear heat source. There is a region of frequencies where both limits are valid, and this provides smooth joining of solutions. The mathematical model gives results for two extreme cases: one and an infinite number of tubes. It is valid for burial depths exceeding 2 m, starting from which the heat power of the system ceases to depend on the power exchange by the ground surface with the surrounding medium. In this case, to calculate a heat exchanger it is necessary to know the thermophysical parameters of the ground and of the nonperturbed temperature of the ground at the depth of the burial of tubes. The latter temperature can be obtained by carrying out field measurements.

In the present work an analytical expression was obtained for the optimal, from the point of view of heat exchange of air with the ground, tube length. The degree of decrease in the efficiency of the heat exchanger on decrease of the spacing between the tubes was calculated, as well as the dependence of the heat power of the system on time during its operation only in winter for the period of 10 years. Also given are the dependences of the heat power of the system on the length and diameter of tubes, depth of their burial, and air flow rate. The results of calculations agree with the experimental data. The procedure developed does not require cumbersome calculations and can be used for working out design recommendations.

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